Rational Functions(2)

Consider the function f defined by $f(x) = \frac{x^2}{x-1}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

1. Determine the domain of definition D of the function f and calculate the limits of f at the boundaries of D.

$x - 1 \neq 0$; $x \neq 1$

$D =]-\infty;1[\cup]1;+\infty[$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x^2}{x} = \lim_{x\to-\infty} x = -\infty$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x = +\infty$$

$$\lim_{x\to1^-} f(x) = \frac{1}{0^-} = -\infty$$

$$\lim_{x\to1^+} f(x) = \frac{1}{0^+} = +\infty$$

So $x = 1$ is a vertical asymptote

Consider the function f defined by $f(x) = \frac{x^2}{x-1}$. (C) its representative curve in an orthonormal system $(O; \vec{i}; \vec{j})$.

2. Show that the line (d) : $y = x + 1$ is an oblique asymptote and study its relative position with respect to (C).

$$f(x) - y = \frac{x^2}{x-1} - (x+1) = \frac{x^2 - x^2 + 1}{x-1} = \frac{1}{x-1}$$

$\lim\limits_{x \to \pm\infty} \frac{1}{x-1} = \lim\limits_{x \to \pm\infty} \frac{1}{x} = \frac{1}{\pm\infty} = 0$ So (d) is an oblique asymptote at $\pm\infty$.

Relative position

$$f(x) - y = \frac{1}{x-1}$$

x		1	
$f(x) - y$	$-$	‖	$+$
Position	(C) is below (d)	‖	(C) is above (d)

Consider the function f defined by $f(x) = \frac{x^2}{x-1}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

3. Show that the point I(1,2) is a center of symmetry.

$a = 1 \ \ ; b = 2$

Domain $D =]-\infty;1[\cup]1;+\infty[$ is centered at 1

$f(2a-x)+f(x) = f(2-x)+f(x) = \frac{(2-x)^2}{2-x-1} + \frac{x^2}{x-1} = \frac{4-4x+x^2}{1-x} + \frac{x^2}{x-1}$

$= \frac{4-4x+x^2}{1-x} - \frac{x^2}{1-x} = \frac{4-4x+x^2-x^2}{1-x} = \frac{4-4x}{1-x} = \frac{4(1-x)}{1-x} = 4 = 2(2)$

So I is a center of symmetry

Consider the function f defined by $f(x) = \frac{x^2}{x-1}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

4. Calculate $f'(x)$ and set up the table of variations of f.

$$f'(x) = \frac{2x(x-1)-1(x^2)}{(x-1)^2}$$

$$= \frac{2x^2-2x-x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

x		0		1		2	
$f'(x)$	+	0	−	||	−	0	+
$f(x)$	-∞ ↗	0	↘ -∞	||	+∞ ↘	4	↗ +∞

$f'(x) = 0$; $x(x-2) = 0$

$x = 0$ or $x - 2 = 0$

$x = 2$

$$f(0) = \frac{0^2}{0-1} = 0$$

$$f(2) = \frac{2^2}{2-1} = 4$$

Consider the function f defined by $f(x) = \frac{x^2}{x-1}$. (C) its representative curve in an orthonormal system $(O; \vec{i}; \vec{j})$.

5. Plot (C).

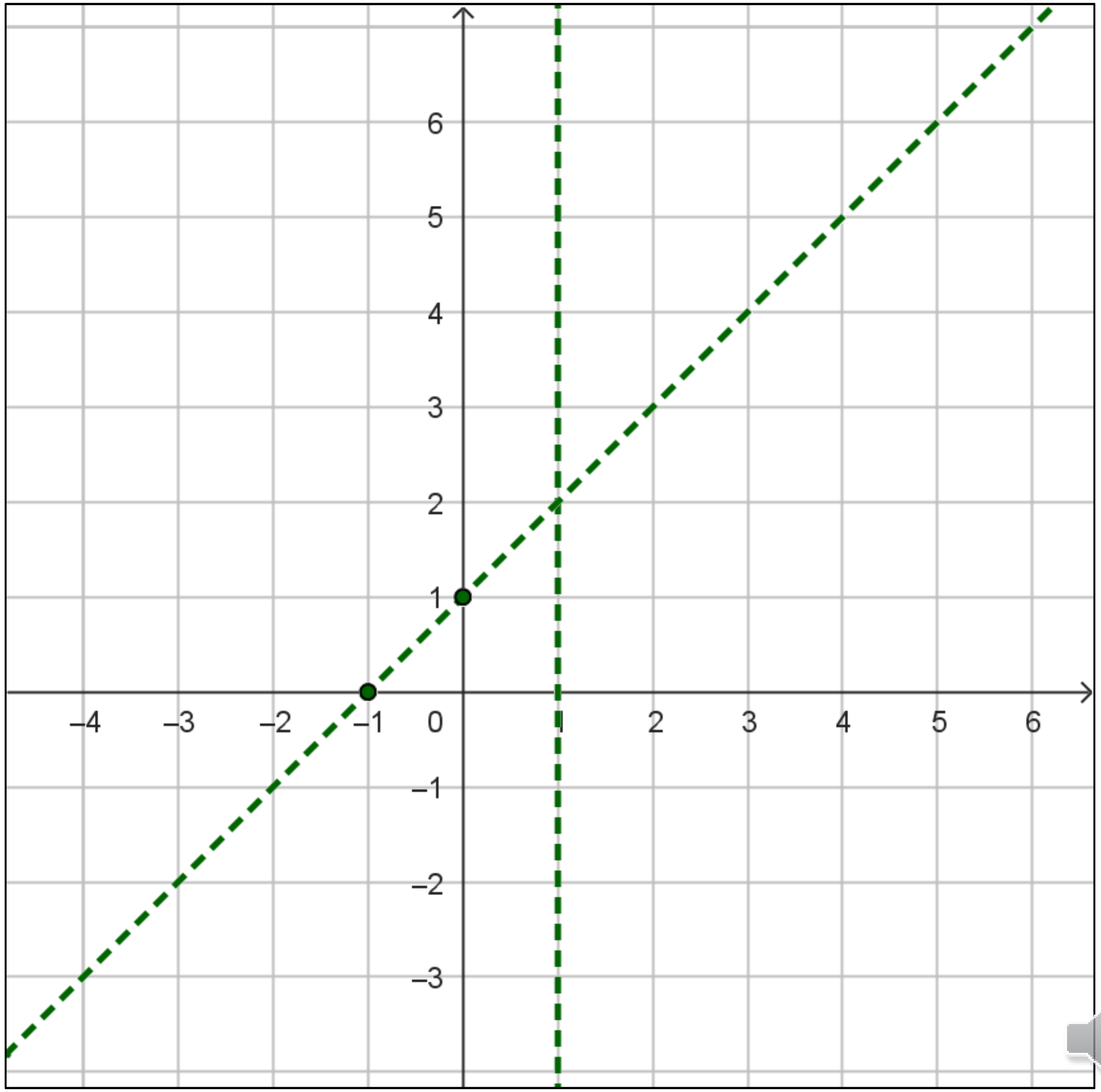
- Start by the asymptotes:

$x = 1$ vertical line

$y = x + 1$ Oblique line

For $x = 0$; $y = 1$

For $x = -1$; $y = 0$



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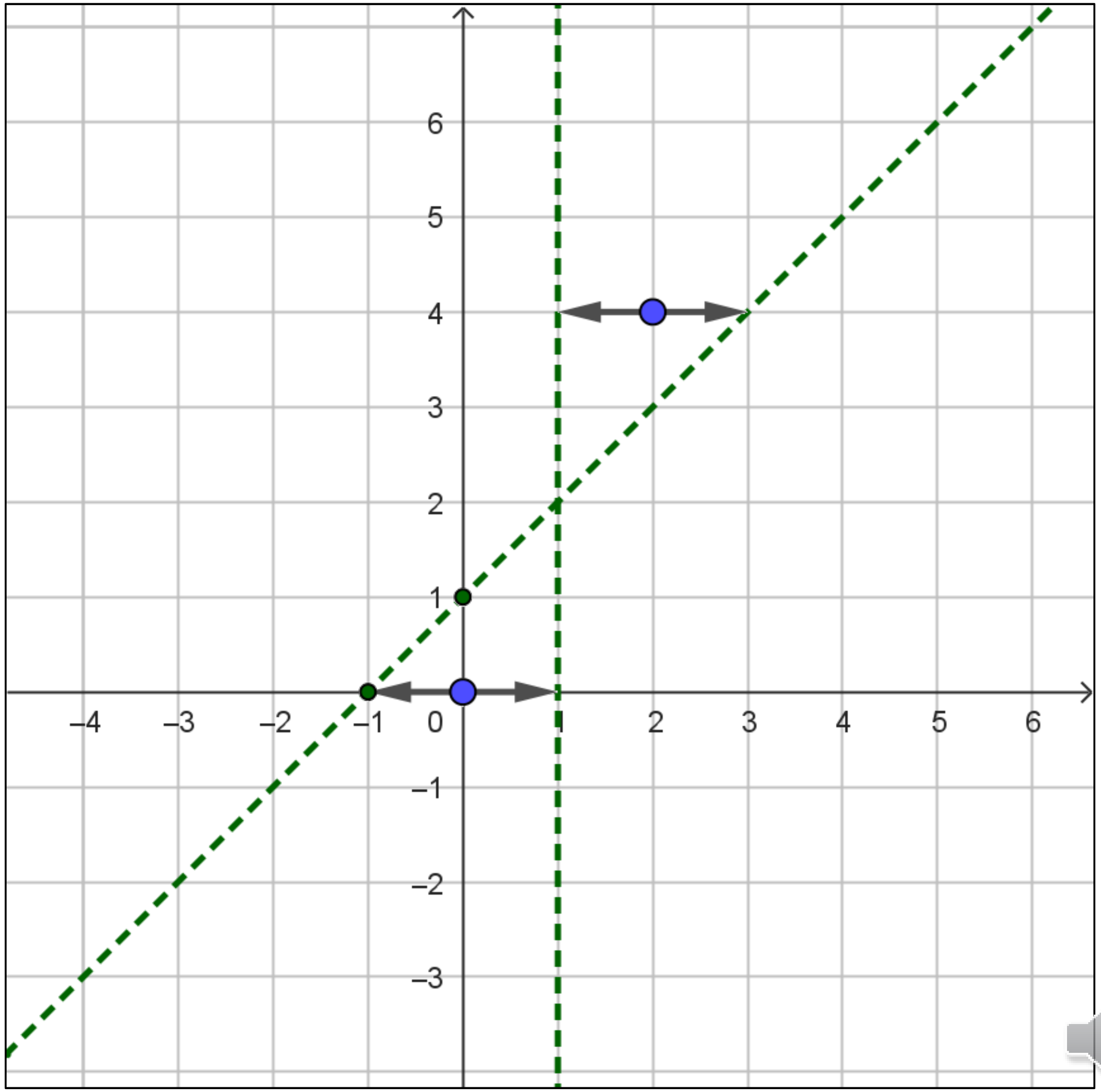
5. Plot (C).

- Place the extrema

x		0		1		2	
$f'(x)$	+	0	−	‖	−	0	+
$f(x)$	-∞ ↗	0	↘ -∞	‖	+∞ ↘	4	↗ +∞

- Particular points:

It is (0;0)



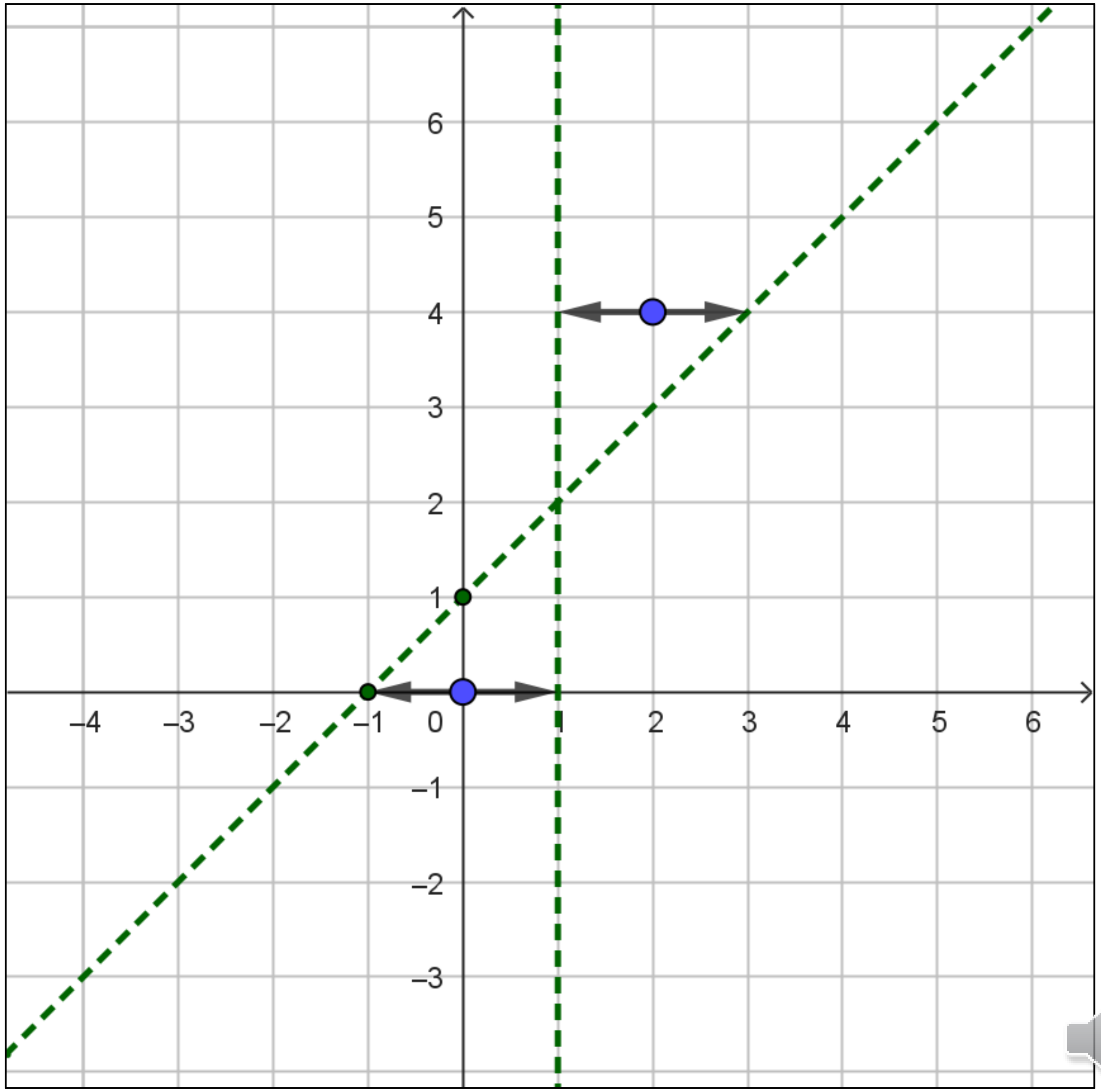
Consider the function f defined by $f(x) = \frac{x^2}{x-1}$. (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

5. Plot (C).

- Start drawing based on the table of variations

x		0		1		2	
$f'(x)$	+	0	−	‖	−	0	+
$f(x)$	-∞ ↗	0	↘ -∞	‖	+∞ ↘	4	↗ +∞

O.A.



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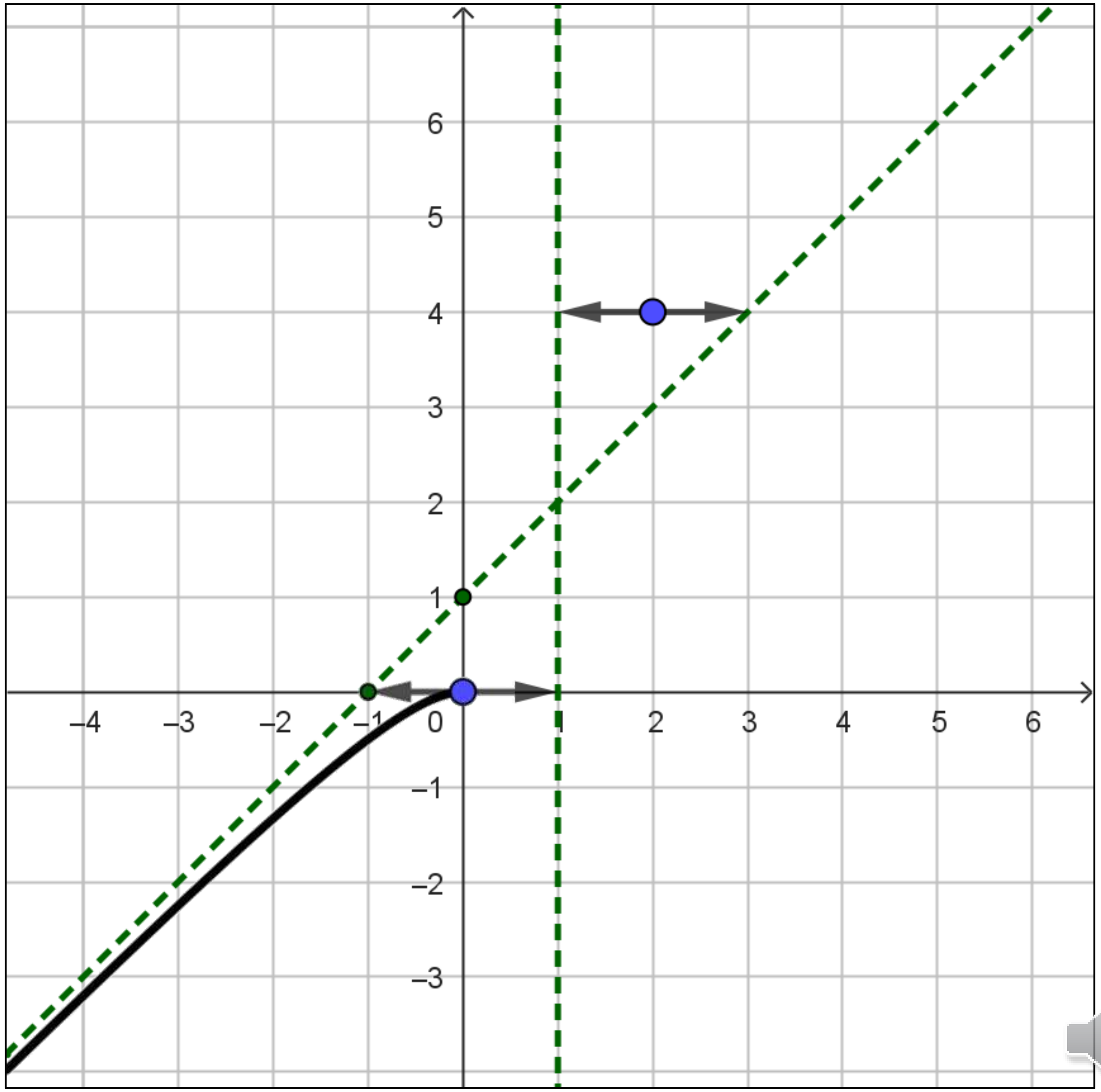
5. Plot (C).

- Start drawing based on the table of variations

x		1	
$f(x) - y$	$-$		$+$
Position	(C) is below (d)		(C) is above (d)



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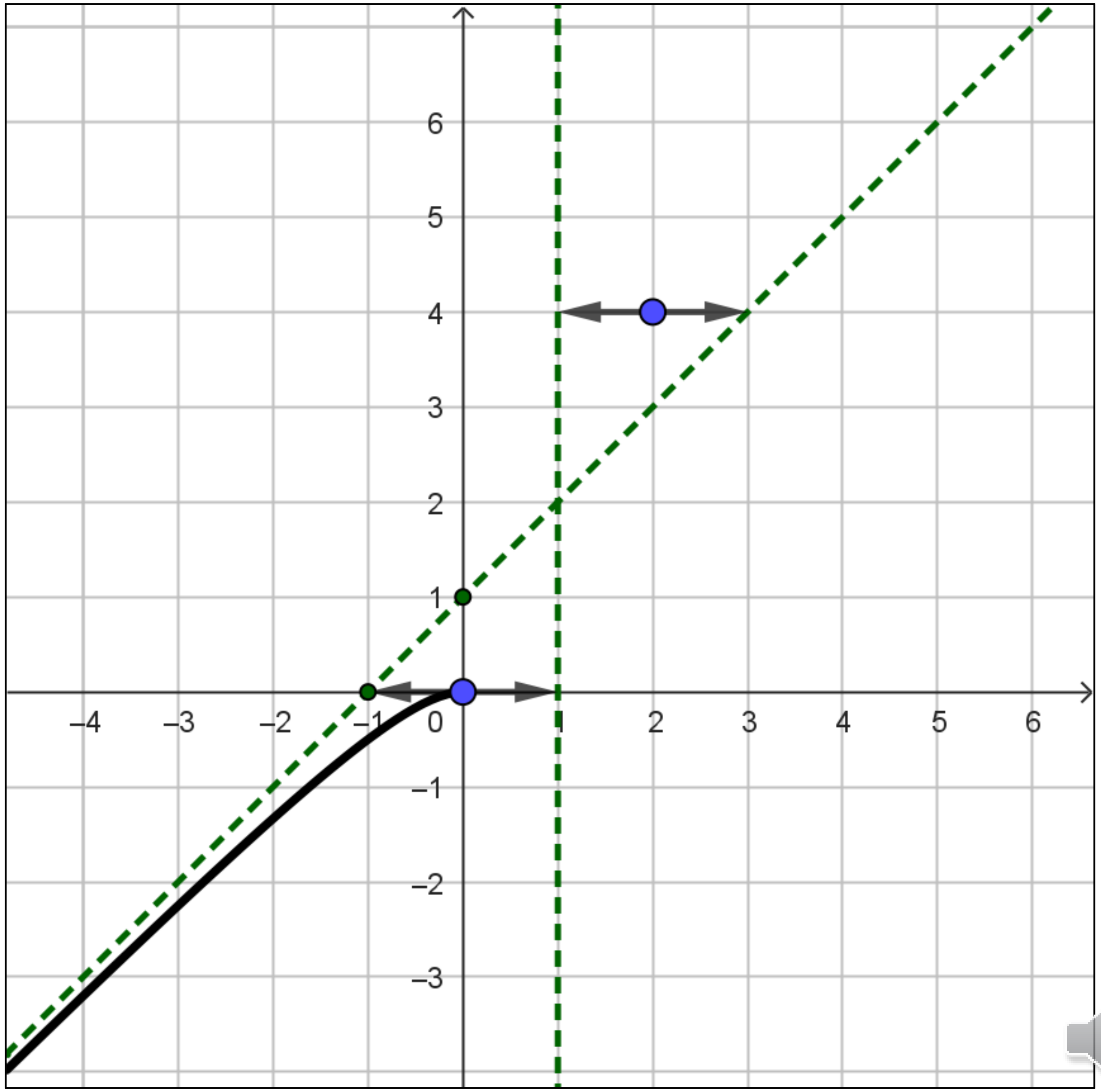
5. Plot (C).

- Start drawing based on the table of variations

x		0		1		2	
$f'(x)$	+	0	−	‖	−	0	+
$f(x)$	$-\infty$ ↗	0	↘ $-\infty$	‖	$+\infty$ ↘	4	↗ $+\infty$

V.A.



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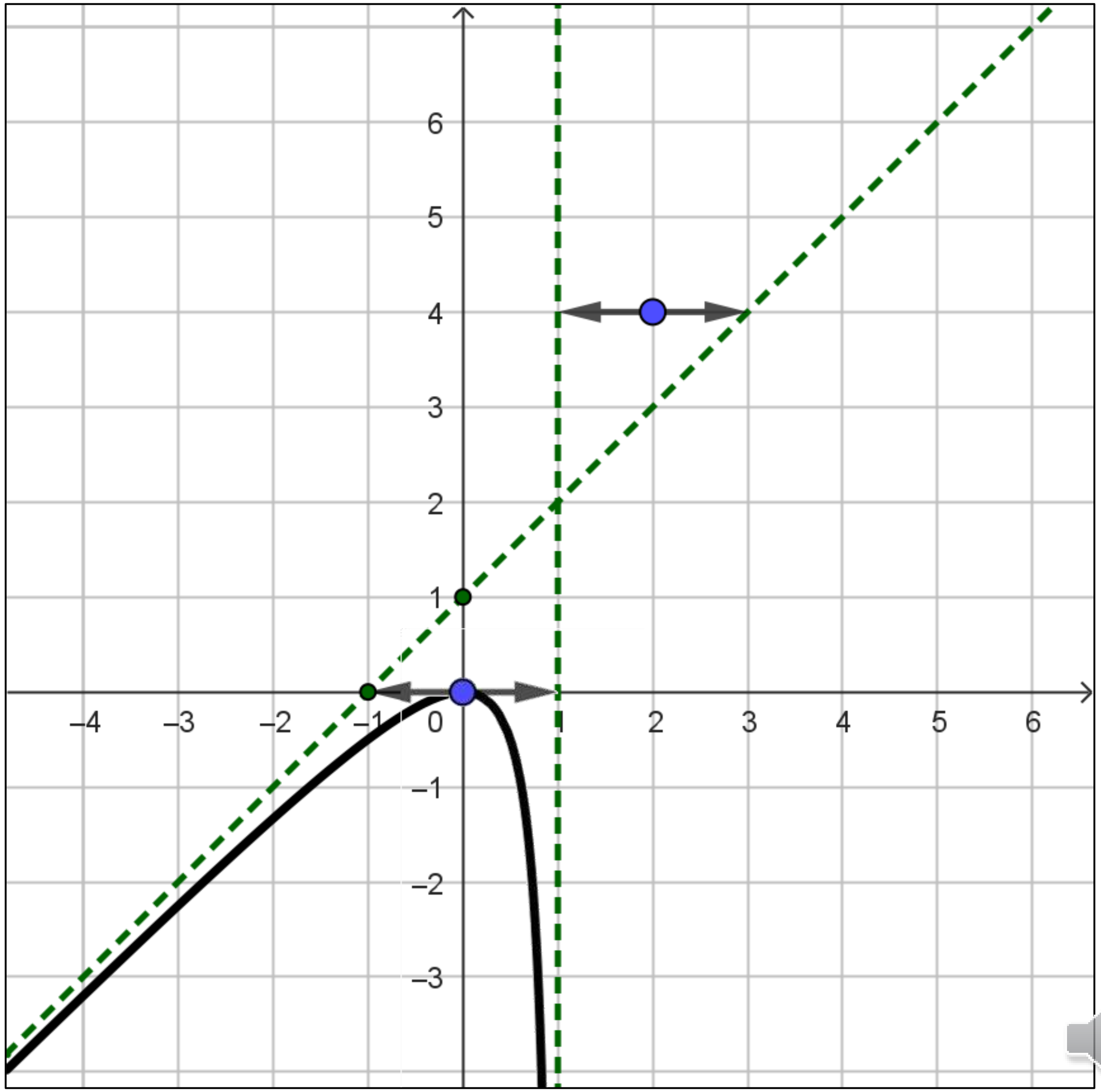
orthonormal system $(O;\vec{i};\vec{j})$.

5. Plot (C).

- Start drawing based on the table of variations

x		0		1		2	
$f'(x)$	+	0	−	‖	−	0	+
$f(x)$	$-\infty$ ↗	0	↘ $-\infty$	‖	$+\infty$ ↘	4	↗ $+\infty$

V.A.



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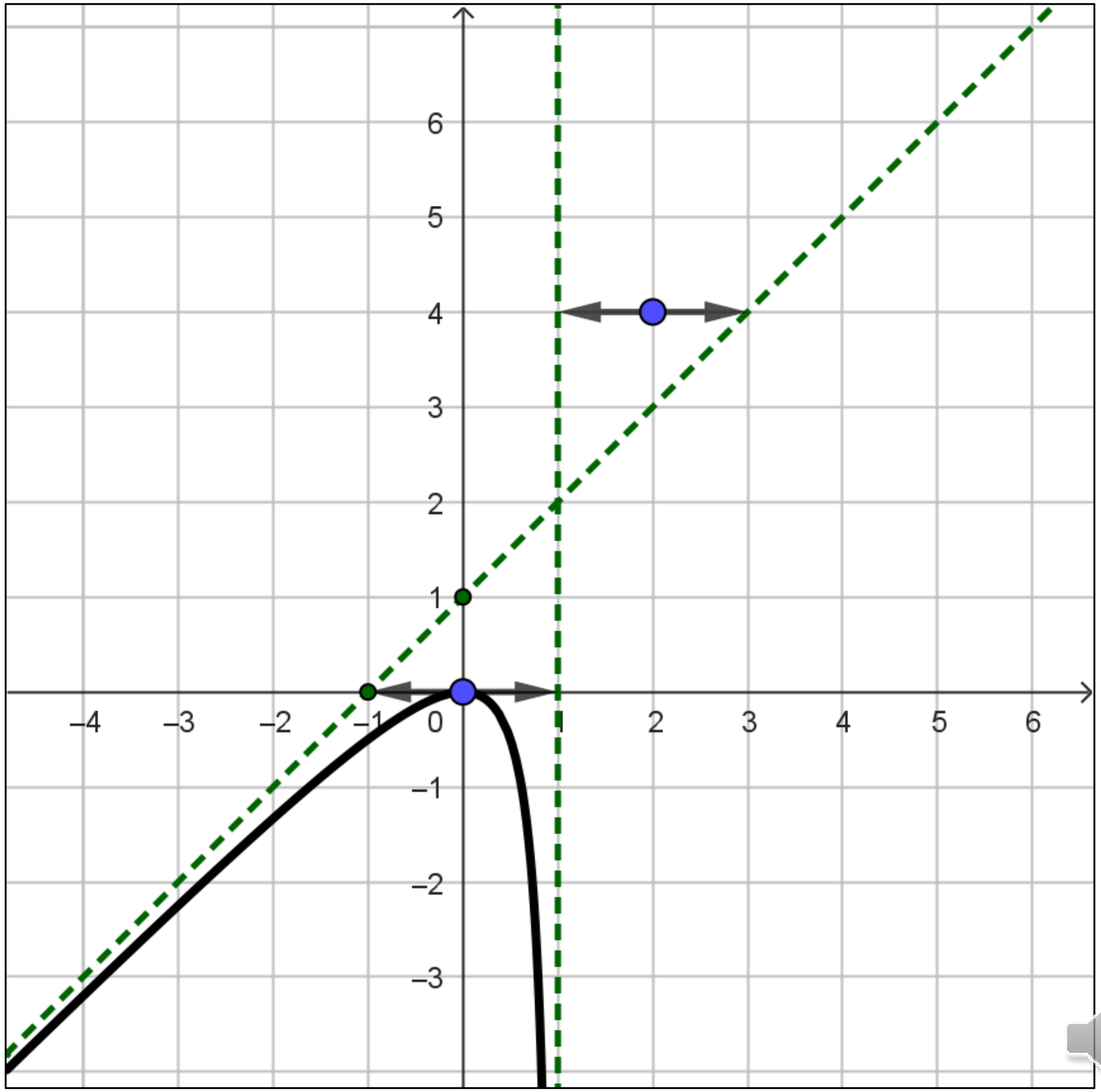
5. Plot (C).

- Start drawing based on the table of variations

x		0		1		2	
$f'(x)$	$+$	0	$-$	‖	$-$	0	$+$
$f(x)$	$-\infty \nearrow$	0	$\searrow -\infty$	‖	$+\infty \searrow$	4	$\nearrow +\infty$

V.A.



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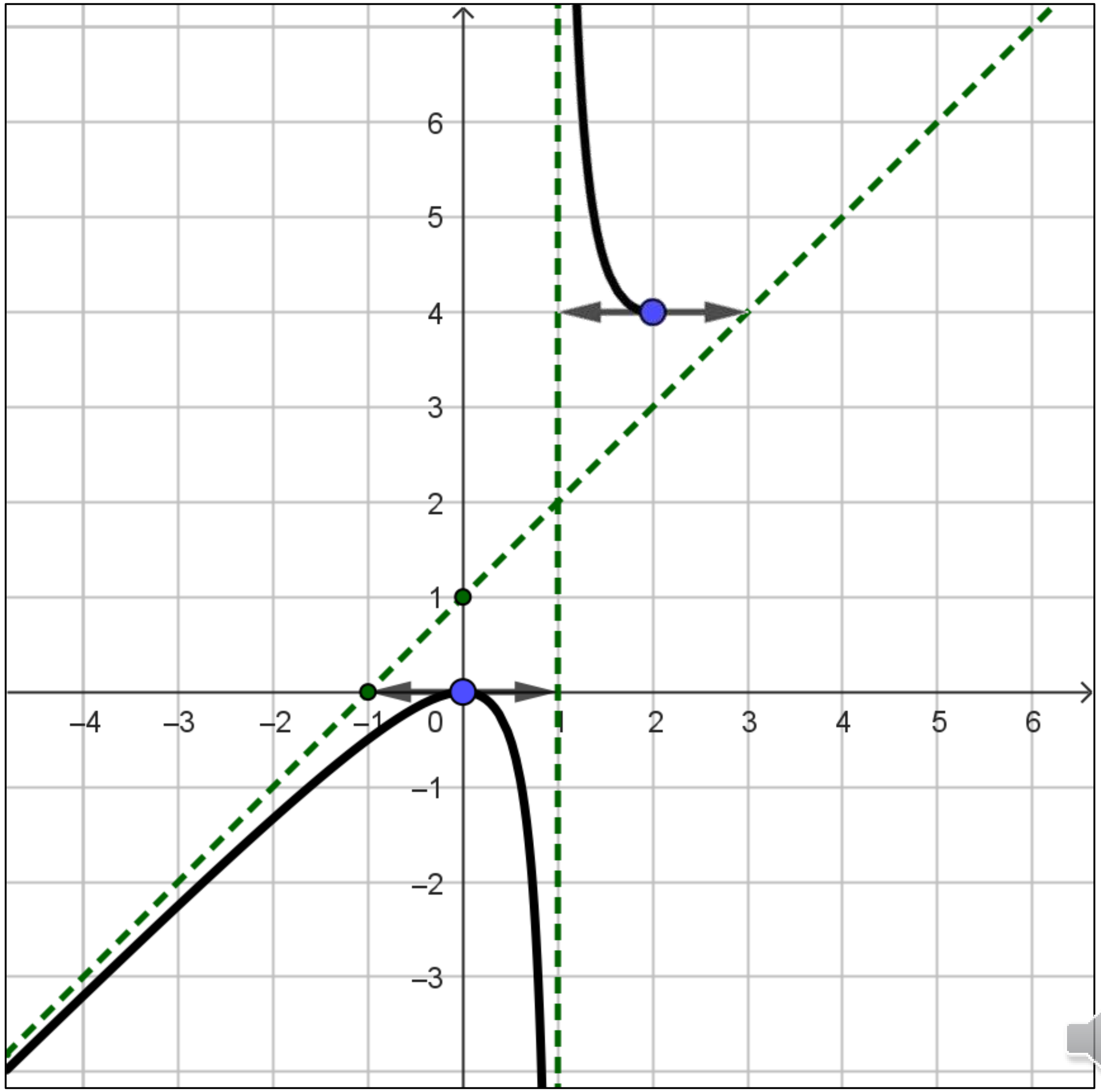
- Start drawing based on the table of variations

x		0		1		2	
$f'(x)$	+	0	−	‖	−	0	+
$f(x)$	$-\infty$ ↗	0	↘ $-\infty$	‖	$+\infty$ ↘	4	↗ $+\infty$

V.A.



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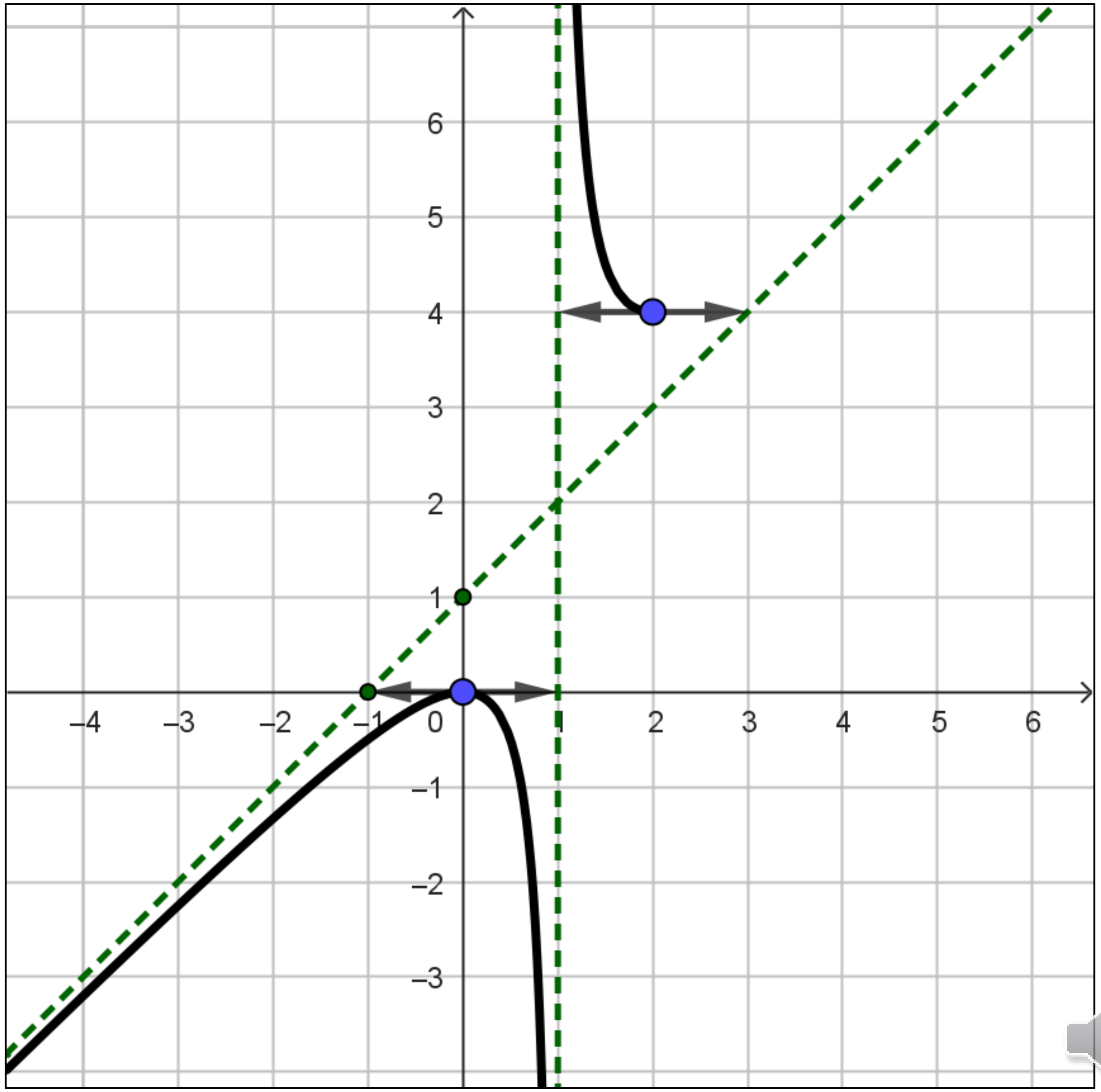
5. Plot (C).

- Start drawing based on the table of variations

x		0		1		2	
$f'(x)$	$+$	0	$-$	‖	$-$	0	$+$
$f(x)$	$-\infty$ ↗	0	↘ $-\infty$	‖	$+\infty$ ↘	4	↗ $+\infty$

O.A.



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5. Plot (C).

➢ Start drawing based on the table of variations

x	0		1		2	
$f'(x)$	+ 0	−	‖	−	0 +	

x		1	
$f(x) - y$	−		+
Position	(C) is below (d)		(C) is above (d)

O.A.